1993-60

GAS FLOW RATES INTO VOIDS BELOW TIMBER FLOORS THEORY AND EXPERIMENT

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ABSTRACT

This paper reports on analytical and experimental studies of the rate of flow of gas into a timber floored test hut at the Building Research Establishment. The analytical results give insight into the nature of the flow through bare soil and will prove useful in verifying computational models. The experimental work took place at the BRE radon pit. The flow rates through the sand produced by different pressures in the under floor space were measured. The analytical solution gives good agreement with the measured flow rate and will in future be compared with the measured pressure field. The results will be useful in understanding the effectiveness of remedial measures for this type of floor structure.

INTRODUCTION

In this paper a comparison is made between analytical methods of predicting the entry of gas into a timber floored building and a measurement of the same flow. The analytical solutions enable the flow to be predicted from the soil permeability, the driving pressure and a constant factor which can fairly easily be calculated from the geometry of the building. A detailed calculation of the pressure field is not needed, but the methods used enable it to be found if desired. The analytical solution gives good agreement with the measured flow rate and will in future be compared with the measured pressure field. The results will be useful in understanding the effectiveness of remedial measures for this type of floor structure.

Timber Floors

Timber is the dominant flooring material in the UK, with over 90% of homes having some part of the floor as suspended timber. This consists of a timber floor suspended on joists above a small air gap, of 10 to 50 cm depth, which should be ventilated, but isn't in all cases. In some houses there is an oversite layer of concrete, but this is often of poor quality.

As a result ~~there~~ is an easy entry path for radon through the soil, into the void and through the many cracks in the timber floor. The methods used in the UK to remediate these houses are discussed in another paper at this conference[1] but ~~mainly~~ involve improving the ventilation of the void.

Why Use Analytical Methods?

The aim is to find exact solutions to as many problems as possible. An analytic solution is often more versatile than a numerical one, since the answer will be in terms of parameters which are more easily varied than those in a numerical solution. They also indicate which problems need to be solved numerically and can help with the validation of numerical models. Usually only a fairly simple geometry can be solved exactly, and any problem without an exact solution will need to be solved numerically. But a simpler approximation to the real problem gives an idea of the type of result to expect.

ANALYTICAL SOLUTIONS

Laplace's Equation is used to model the pressure in soil wherever the Darcy Law for gas flow is valid. Recent work at Lawrence Berkeley Laboratory has shown that soil gas flow is not always linear[2]. However this generally happens when there is a sub-slab ventilation system being used (or radon sump in the UK), and gas flow velocities become high, of order 0.1 ms^{-1} or more. The problems considered here have velocities of order 10^{-4} ms^{-1}, so the Darcy Law remains valid.

The key equations used are Darcy's Law:

$$Q = -k/\mu . A . dP/dx \qquad ...(1)$$

where Q is the flow rate (m^3h^{-1})
k is the permeability of the soil (m^2)
μ is the viscosity of the fluid flowing (Pa.s)
A is the area of flow (m^2)
P is the excess pressure of the fluid compared to ambient (Pa)
x is the length over which flow occurs (m)

and combining this with the continuity equation gives Laplace's Equation[3]:

$$\nabla^2 P = 0 \qquad ...(2)$$

This equation describes the pressure field within a region of soil. If it can be solved, then the flow rate can be found from Darcy's Law. From the flow rate a prediction of a likely radon entry rate can be made, by assuming the soil radon concentration. The worst case assumption is that the deep soil radon concentration applies at all levels.

In the following examples problems of increasing complexity are solved using different techniques. It is not possible to do more than indicate the method of solution here; contact the author for more details.

Solution 1: Sloping Step Problem

The first problem represents a simple two dimensional building, with a fixed pressure inside and out, and a linear change in pressure across the walls. The parameters n and m are used to allow a general solution to be found. It is defined by the following pressure distribution on y=0, also shown as figure 1.

$$\begin{array}{lll} |x| > n, & P(x,0) = 0 & \\ m < |x| < n & P(x,0) = n - |x| & ...(3) \\ |x| < m & P(x,0) = n - m & \end{array}$$

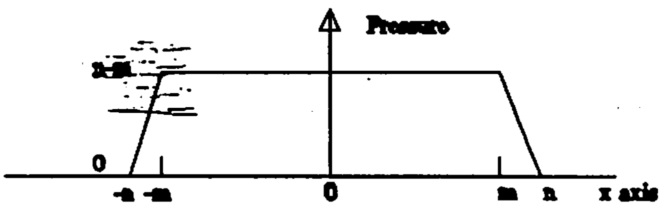

Figure 1: Pressure Distribution On y=0 For Solution 1

The value of m represents the distance from the centre of the house to the inside edge of the wall, while n is the length to the outer edge of the wall. Using a Laplace transform method the solution is found to be

$$\pi P(x,y) = \left((n\pm x)\cdot \tan^{-1}\left(\frac{n\pm x}{y}\right)\right) - \left(\frac{y}{2}\cdot \ln(y^2 + (n\pm x)^2)\right) \qquad ...(4)$$

$$- \left((m\pm x)\cdot \left(\tan^{-1}\left(\frac{m\pm x}{y}\right)\right) - \left(\frac{y}{2}\cdot \ln(y^2 + (m\pm x)^2)\right)\right)$$

Each of the four braced terms are to be repeated with plus and then minus, giving eight terms in all. This result for the pressure field is plotted in figure 2, with the factors m and n equal to 2 and 3 respectively, and the pressures normalised. This shows that the boundary conditions have been satisfied correctly, with a linear pressure change between m and n on the x axis.

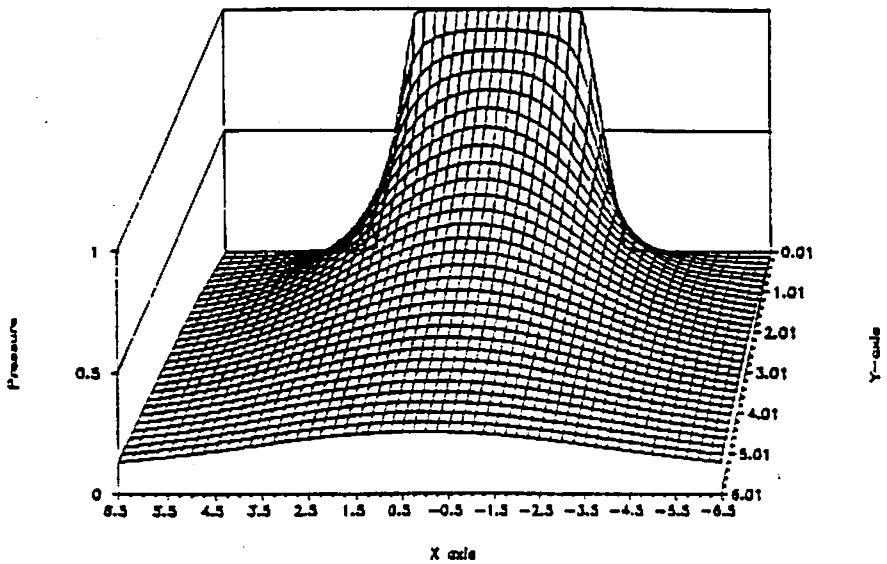

Figure 2: Pressure contours for solution 1

Flow Rate

By differentiating (4) to give dP/dx, and then integrating this in Darcy's Law (1) with respect to x from -m to +m, gives:

$$Q = \frac{2kP_0}{(n-m)\pi\mu}\cdot[(n+m)\ln(n+m) - (n-m)\ln(n-m) - 2m\ln(2m)] \qquad ...(5)$$

Here the indoor pressure is set to P_0, and the factor (n-m) is needed to account for this. Note that the flow is predicted to be linearly related to the indoor pressure, the permeability of the soil and a factor relating to the shape of the building.

This flow result is the same as that produced by Landman and Delsante[4] but via an alternative method involving Fourier series, although they gave their result in terms of different parameters. Note that in the limit m tends to n, the first and third terms in the square bracket cancel and the second term simplifies to ln(n-m). This predicts an infinite flow rate, as can be found for the simpler 'Top Hat' problem.

Solution 2: Mixed Boundary Condition

In this ~~problem~~ there are thick walls which do not extend into the ground, through which no flow is assumed to occur. ~~The~~ pressure is assumed fixed at P_0 inside the house, and at 0 outside. The conditions on y=0 are shown on ~~figure 2.1~~ below. Any similar house would give the same result scaled up or down, so the inside of the wall is set to be at 1, and the outside of the wall at m.

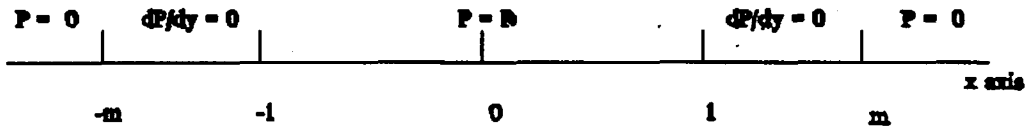

Figure 3: The pressure field on y=0 for solution 2

The method used to solve this problem is not straightforward, and is taken from[5]. It uses the fact that any differentiable function of the complex variable z = x + iy is a solution to Laplace's equation, and the relationship between the real and imaginary parts of a function of a complex variable. In this paper i is the square root of -1.

The solution to the pressure field is described by a function V, while the gradient of the pressure is described by another function U. These two are combined to give the complex function W = U + iV, and the solution to W can be found. From it U and V are then obtained. The boundary conditions for the W problem are shown below. One of U and V must be defined everywhere along y=0.

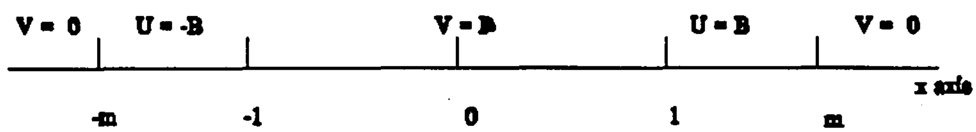

Figure 4: Boundary conditions on the complex function W

B is a constant to be determined. Finding B is a major part in finding the solution to the problem. B is also important in finding the flow rate into the house. After solving for W the complex part of W is taken, which will give the pressure field V(x,y).

Using[5] the solution is written as

$$W = \frac{1}{\pi i}\cdot\left(\frac{(z+m)(z-1)}{(z+1)(z-m)}\right)^{\frac{1}{2}}\cdot\int_{-m}^{m}\left(\frac{(t-m)(t+1)}{(t-1)(t+m)}\right)^{\frac{1}{2}}\cdot\frac{h(t)}{t-z}\cdot dt + C \qquad ...(6)$$

This must be expanded term by term to include the form of h(t), and the sign and nature of the root terms. h(t) is -B for -m < x < -1, i.Po for -1 < x < 1 and +B for 1 < x < m.

By applying the boundary conditions we can find the constants B and C. They come out to be:

$$B = -P_0 \cdot \frac{\displaystyle\int_0^1 \frac{dt}{\sqrt{[(1-t^2)(m^2-t^2)]}}}{\displaystyle\int_1^m \frac{dt}{\sqrt{[(t^2-1)(m^2-t^2)]}}} \qquad C = 0 \qquad \ldots(7)$$

m	$-B/P_0$
1.1	1.4101
1.2	1.20
1.5	0.95
2.0	0.7817

Table 1: Equations for B and C and values of B for varying parameter m

Both of the integrals in the expression for B are elliptical integrals, which can be found from tables, e.g.[6].

Numerical Integration

The expression for W is not integrable exactly because of the elliptical integrals in it. Hence to obtain the solution a numerical integration using Simpson's Rule was necessary to produce figure 5. This gives the expected form of result, and meets the boundary conditions. In this case the parameter m had value 2, which is an unrealistic situation, but shows the boundary conditions better than an example with smaller m.

In figure 5 the whole region is shown as a pressure contour plot. The flow of gas would be perpendicular to the pressure contours at all points, with the rate proportional to the pressure gradient or separation of the lines. The non linear pressure drop between x=1 and x=2 is the main difference between this result and solution 1 where the pressure drop across the wall was forced to be linear.

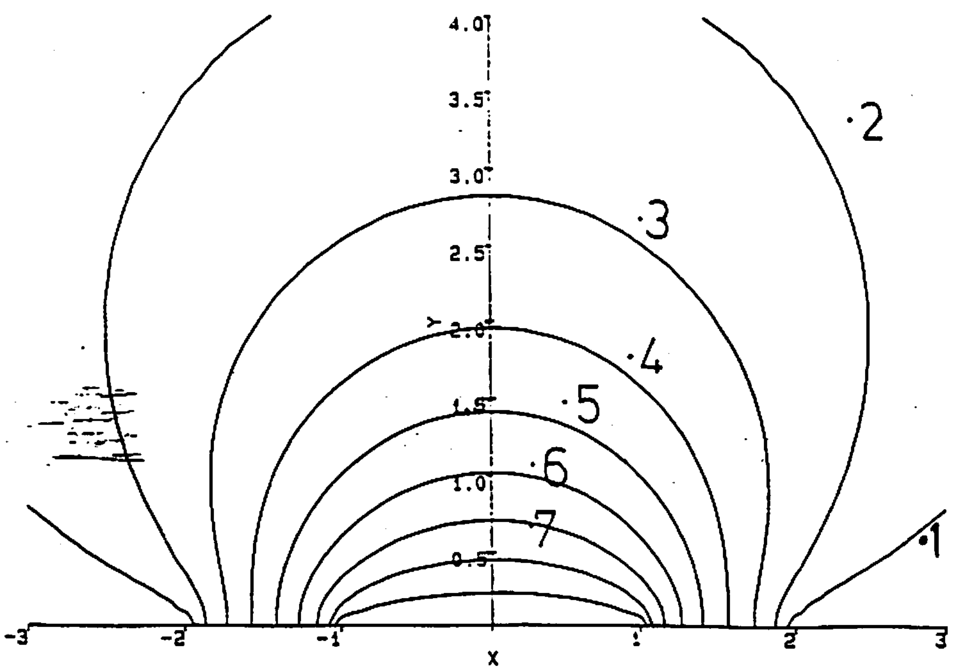

Figure 5: Pressure field for solution 2

Flow Produced By The Pressure Distribution

A key result is the flow rate into a house produced by a given pressure distribution. This is given, assuming linear i.e. Darcy flow (1), by the integral of the pressure gradient between the two walls. That is:

$$Flow = \int_{-1}^{1} -\frac{k}{\mu} \cdot \frac{\delta P}{\delta y}\Big|_{y=0} \cdot dx \qquad ...(8)$$

But this simplifies to give

$$Flow = -2B \cdot k/\mu \qquad ...(9)$$

The values of B for given values of P_o and m are given in table 1 above. The flow rates predicted by the different results are discussed later.

<u>*Solution 3: Mixed Boundary Problem With Depth*</u>

Conformal Mapping is a process whereby a solution to a problem found in one co-ordinate set is transferred to another co-ordinate set. This then gives a solution to a problem which may not have been soluble in another way, or may be simpler than a different direct method.

Real walls have foundations in the soil, which cannot be ignored in gas flow. As a first step consider a thin wall extending down into the ground as shown in figure 6 below. Note that d and c are positive, real numbers.

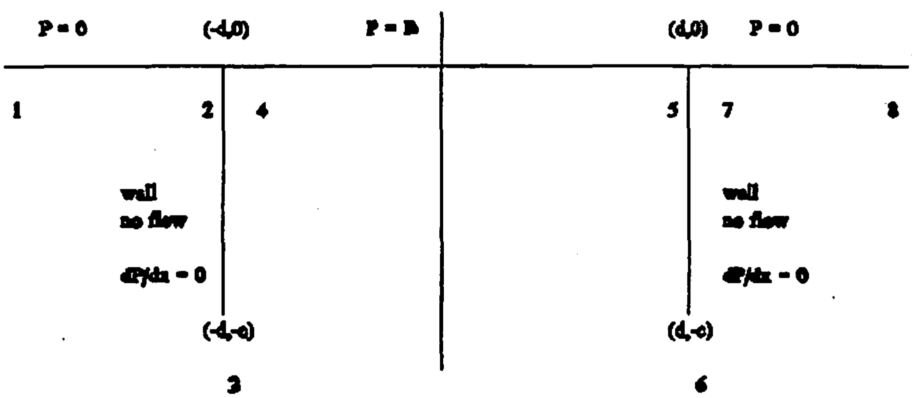

Figure 6: Diagram for the mapped pressure field problem, the Z(x,y) plane

The task is to find the transformation that maps points from figure 6 onto figure 3 of solution 2, so that the pressure field for problem 3 can be found without further solving of the basic equations. Unfortunately the transformation is not easy to find. The layout we are trying to map onto is shown in figure 7. Here the points 1 to 8 from figure 6 are shown transformed to the W(X,Y) plane as points 1' to 8'.

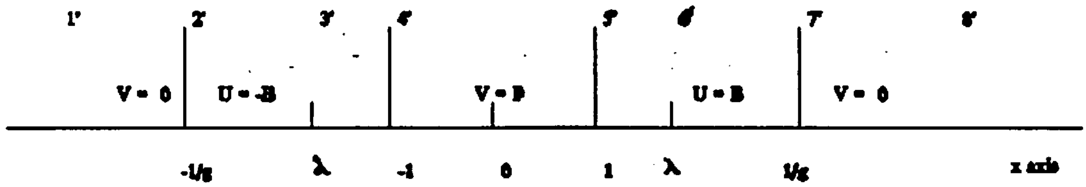

Figure 7: Diagram of the plane mapped onto, the W(X,Y) plane.

The method used is called the Schwarz-Christoffel transformation[7]. It gives the transformation which maps any closed polygon onto a plane, if the positions of the corners and angles at each corner are known. It is:

$$Z = f(W') = a \int_{W_0}^{W'} \sum_{j=1}^{n} (W-e_j)^{(\alpha_j/\pi)-1}\, dW + b \qquad \text{...(10)}$$

where

a, b and W_o are constants to be determined
α_j are the angles at the points in the Z plane
e_j are the positions of the corresponding points in the W plane
n is the number of points on the x axis, here 7 since points 1 and 8 are the same

In this case the angles α_j are $\pi/2$, 2π, $\pi/2$, $\pi/2$, 2π, $\pi/2$ respectively for the points 2 to 7. The e_j are the points on the Y axis $-\infty$, $-1/g$, $-\lambda$, -1, 1, λ, $1/g$, ∞.

Hence the transformation needed is:

$$Z = \frac{a}{g^2} \cdot \left(\int_0^{W'} \frac{\sqrt{(g^2W^2-1)}}{\sqrt{(W^2-1)}} \cdot dW + (1-g^2\lambda^2) \cdot \int_0^{W'} \frac{dW}{\sqrt{[(g^2W^2-1).(W^2-1)]}} \right) \qquad \text{...(11)}$$

This expression contains commonly occurring elliptical integrals which have to calculated numerically for nearly all values of g and W'. The values are found in tables, for example[6], or by numerical integration.

Finding the parameters in the transformation

There are ~~now~~ three unknowns to find; a, λ and g. These can be found from the points where the result of the transformation ~~is~~ known; (d,0), (d,-c) and (d,0) again. From these it is possible to find the values of the parameters g and λ ~~from~~ c and d. Some values are given below.

1/g	λ	c	d	a
3	1.991	1	1.8	0.322
2	1.499	1	2.87	0.986
1.5	1.2499	1	4.926	2.654
1.2	1.1	1	10.969	8.328

Table 2: Parameters in solution 3

Hence λ is very close to halfway between 1 and 1/g, but not exactly equal to it. Expressions can be found for λ when 1/g is close to 1 and when it becomes large, but these are not included here.

The result from the numerical integration of the transformation (2.5) is Figure 8, where 1/g was 2. Each point calculated from solution 2 earlier was transformed to the corresponding point for this problem, and its pressure value plotted against the new co-ordinates. Figure 8 shows a good result, of the form we would have expected. The curved line at the bottom shows the limit to the transformed domain; there is no reason why the plot could not be extended further. The 'untidy' region below x = 2.87 is due to the plotting package having difficulty with the discontinuity in the data.

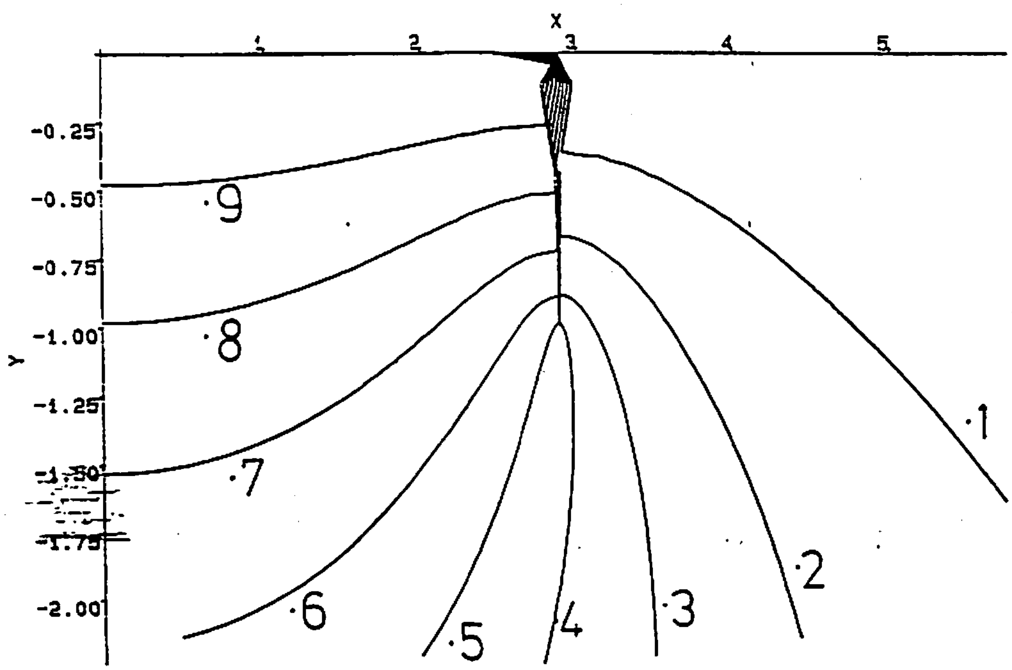

Figure 8: Pressure field contours for solution 3

Flow rate

Integrating Darcy's Law (1) as before gives the flow rate to be exactly the same as for the untransformed problem. Hence the flow is given by:

$$\text{flow} = k/\mu.2B.P_o \qquad \ldots(12)$$

where B was found earlier. It is a constant for any given problem in the 'flat', untransformed co-ordinates. This flow is the same as for the corresponding problem and means a given value of g in the 'flat' problem corresponds to a specific ratio of c to d in the second problem.

The same method has been taken a stage further to allow a thick wall to be transformed in the same way. To date the pressure field has not been calculated, but the flow rates have been found and are used later to compare with the experimental results. This is achieved by using the results for the points where the results of the transformation are known to find values for the dimensions of a sample wall which correspond to the same flow rate as a known solution for the 'flat' solution 2. The details will be published later.

Experimental results- The BRE Radon Pit

The radon pit is an unusual piece of experimental equipment. It consists of sand about 6m by 10m and 4m deep. The sand contains high levels of radium, which results in high radon levels in soil gas in the sand. On top of the sand two structures have been built. These are essentially identical, so that changes to one structure can be monitored against the other as a control. BRE is using the facility to investigate the methods to remediate houses with timber floors, but this paper does not report on these experiments.

In this test the flow through the soil due to an applied pressure below the floor was measured, since this is the most useful prediction from the theory. The soil leakage can be measured by balancing the pressure in the 'house', P_m, with that in the under floor area, P_u. The idea is shown in the diagram below.

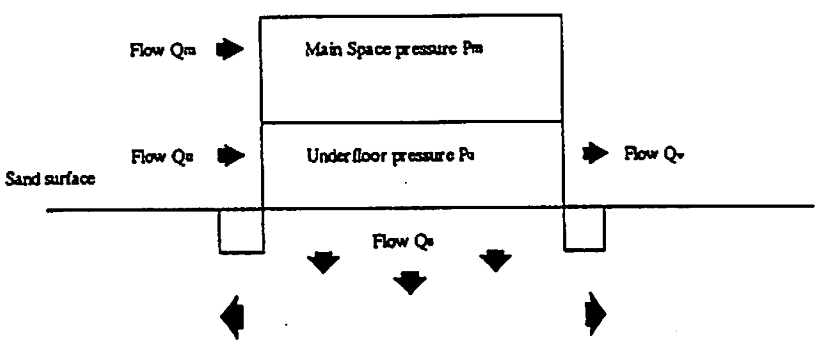

Figure 9: The experimental arrangement

The pressure across the floor is balanced using two fans. One fan blows air into the main space, at a rate Q_m. A second fan blows into the under floor space, at a rate Q_u. All the visible holes in the wall are sealed. The flow Q_u, when the pressures are equal across the floor is the 'leakage' of the soil Q_s and the subfloor walls Q_w combined. The walls here were painted on the inside with a bituminous paint, and the air bricks were carefully sealed. As a result the majority of the flow was probably going through the soil and not through the walls.

Hut Fan Flow m^3h^{-1}	Hut-Outside Pressure Pa	Hut-Underfloor Pressure Pa	Underfloor Fan Flow m^3h^{-1}
40	-90	< 0.05	27
36	-70	< 0.02	21
28.5	-55	< 0.02	20
16.5	-28	< 0.02	12

Table 3: Results from the balanced pressure test

These data indicate that the upper part of the house is leakier than the lower part. The data for the underfloor fan fit to the expression

$$Q = 1.4\,(\Delta P)^{0.66}\ m^3h^{-1} \qquad \text{...(13)}$$

and the flow at 50 Pa is predicted to be:

$$Q_{50} = 18.9\ m^3h^{-1}.$$

For the main space fan the 50 Pa flow was:

$$Q_{50} = 26.3\ m^3h^{-1}$$

Hence the overall hut leakage was around 45 m^3h^{-1} or 2 ach (air changes per hour) at 50 Pa. This is not characteristic of UK housing as it much lower than the average of around 15 ach. Another test showed that the floor of the hut had extremely small leakage.

During the test some pressures in the sand were also measured. However more measurements are planned for a later date to improve the spread of data, and to enable a comparison with the predictions of the theory for the pressure field.

Laboratory measurement of permeability

In the laboratory a sample of the sand was used to measure the permeability of the radon pit sand, by blowing air through the sample, and measuring the flow rate and pressure drop producing it. Although this test could be improved in accuracy, and the varying effects of compaction and water content investigated, the permeability was found to be $1\times10^{-10}\ m^2$. This result was confirmed by an earlier similar test performed by the National Radiological Protection Board (NRPB).

COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

The radon ~~pit~~ huts are sitting on a ring beam foundation which has an outside dimension of 3.3 m, thickness of 0.58 m and depth 0.4 m. Using the conformal mapping technique of solution 3 this gives a flow equivalent to that for a wall with no depth but of thickness 2.48 times the half width of the living space. This means the parameter m of figure 3 is 2.48, which gives a value of B of 0.6 from equation (7).

Hence using this data in equation (9), with the internal pressure P_o = 50 Pa, and the calculated B = 0.6:

$$\text{Flow} = 3.28\times10^{6} \cdot k \qquad (m^3s^{-1} \text{ per metre of wall}) \qquad \ldots(14)$$

Up to this point the flow calculation is fairly precise, but the effect on the flow rate of the corners of the building are hard to include accurately, although these will increase the flow rate above that predicted here. There are 2.14 metres of internal wall in each direction, so neglecting the corner effects the effective length of wall is 4.28 in equation (14):

$$\begin{aligned}\text{Flow} &= 14\times10^{6} \cdot k \qquad (m^3s^{-1})\\ &= 5\times10^{10} \cdot k \qquad (m^3h^{-1}) \qquad \ldots(15)\end{aligned}$$

Since the measured flow rate was 19 m^3h^{-1} this suggests the permeability of the sand is around 4 x 10^{-10} m^2. This is typical for sands[3] and is closeto that measured for the sand in the laboratory, 1 x 10^{-10} m^2. The cause of the difference is likely to be one of the following:

1) Neglecting the corner effects in calculating the theoretical flow rate into the hut
2) The leakiness of the subfloor walls of the hut
3) Leaks from the pipes used to measure the sand permeability in the laboratory
4) Uncertainty in the compaction and water content of the sand in the laboratory

Both 1 and 2 would cause the calculated permeability to be reduced from the 4 x 10^{-10} m^2 predicted, while 3 would similarly reduce the laboratory measured permeability. Point 4 could affect the permeability in either direction, and deserves further investigation. The result is clearly very encouraging, and shows good agreement between the two methods of finding the permeability, well within the considerable experimental errors involved in the experiments.

CONCLUSIONS

In this paper the results of analytic studies of the flow of gas into the void below a suspended timber floor have been presented. These give complicated expressions for the pressure fields being produced, but much simpler forms for the flow rates. In each case the flow rate was found to be proportional to the permeability and the internal pressure as expected, but also proportional to a geometrical factor which can be found comparatively easily for any floor geometry.

A method for measuring the flow rate through the soil below a building with no concrete oversite was described, and the initial results presented. The technique will not be generally applicable because of the leakiness of most buildings, but could be of some use in measuring soil leakages.

The results for the two parts of the work have been compared through the permeability they predict for the sand at the BRE radon pit. Considering the considerable variability of permeabilities and the difficulty of measuring them accurately, the two predictions compare well with a direct experimental measurement of permeability.

In future work the pressure field in the sand will be compared directly with the model predictions, and the analytical method applied to other geometries. The analytical results will be used to look at the effectiveness of radon remedial measures for this type of floor construction.

ACKNOWLEDGEMENTS

The author acknowledges the assistance on the mathematics from his supervisor at University College London, Prof. Susan Brown, and for the experiments from his colleague Paul Welsh at BRE.

REFERENCES

[1] Welsh P, 'Radon: remediation of dwellings constructed with suspended timber floors', to be presented at the AARST 1993 International Radon Conference, Denver, USA, 20-23 September 1993.

[2] Bonnefous Y, Gadgil A et al, '.Field study and numerical simulation of subslab ventilation systems', Environ. Sci. Technol, 1992, 26, pp1752-1759, published by American Chemical Society.

[3] Mowris, R.J. "Analytical and numerical models for estimating the effect of exhaust ventilation on radon entry in houses with basements or crawl spaces". (M.S. Thesis). LBL-22067, Lawrence Berkeley Laboratory.

[4] K A Landman and A E Delsante, 'Steady state heat losses from a building floor slab with vertical edge insulation - 1', Building and Environment, Vol 21, no 3/4, pp177-182, Pergamon Journal Ltd, UK, 1986.

[5] Muskhelishvili, "Singular Integral Equations", Published by P. Noordhoff N.V. Groningen, Holland, 2nd Edition, 1946.

[6] Abramowitz and Stegun, 'Handbook of Mathematical Functions', Dover Publications Incorporated, New York, first published 1965, 9th printing 1970, ISBN 0-486-61272-4.

[7] Carrier, Crook and Pearson, 'Functions of a complex variable: Theory and Technique', Published by McGraw Hill, 1966.