A MATHEMATICAL APPROACH FOR PREDICTING LONG-TERM INDOOR RADON CONCENTRATIONS FROM SHORT-TERM MEASUREMENTS

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ABSTRACT

Assessment of long-term indoor radon concentrations from short-term testing results are of significant importance when radon risk assessment and liability issues are considered. These issues are based on the figure of annual exposure to indoor radon while radon measurement practices are mainly followed the EPA short-term testing protocols. A mathematical framework has been developed that facilitates non-statistical approach to construct the relationship between the short-term indoor radon measurements and the long-term annual indoor radon levels. This approach was based on the application of the time-dependent indoor radon concentrations calculated from the corresponding contributions of indoor radon driving forces for different time periods having a reference starting time. The approach utilizes an analytical procedure that is based on the solutions of the mass balance equation for the radon gas in the indoor environment. The solutions are applied through semi-analytical modeling of time-dependent indoor radon concentrations. This treatment provides a powerful tool and procedure to assess long-term indoor radon concentrations from short-term testing results.

INTRODUCTION

Prediction of long-term indoor radon concentrations has been a major source of disagreement when some issues of the indoor radon problem are considered. The conflict arises from what is considered practical for testing and the health hazards and liability issues. On one hand, obtaining a long-term measurement of the average indoor radon concentration in a structure by continuous testing for possibly several years (a minimum of one year) is not practical (although it is possible). On the other hand, health and some legal-related actions are based on the annual exposure to indoor radon concentration.

Many of the radon testing utilized commercially are performed over a very short period of time, typically 48 hours, following the US Environmental Protection Agency short-term testing protocols. Although such testing may be sufficient for determining the need for mitigation action for some cases, using these results to represent the annual indoor radon concentration in the structure can create major debate for marginal cases, estimation of health hazards, and liability related issues. The problem is particularly complex given the known fluctuations of indoor radon concentrations.

Several studies have addressed the issue of predicting annual indoor radon concentration from short-term testing results. Such studies utilized two basic methodologies in approaching the problem. The first methodology uses specific research structures and develops correlations between indoor radon concentrations and other environmental parameters that are simultaneously monitored for a long period of time (a minimum of one year). Then researchers experimentally fit the data and obtain the best representation of indoor radon concentration based on the particular set of data (Hull and Reddy 1990; Reddy et. al. 1990).

The second methodology analyzes large data bases (survey data) related to indoor radon measurement, although they are not particularly designed for this purpose, and tries to obtain general correlations relating indoor radon concentrations to the available parameters in the survey data set (White et. al 1994, White et. al. 1992; Cohen 1990, Roessler et. al. 1990, Steck 1990;). The second approach covers a large number of houses, but there is uncertainty in the results and a lack of experimental control parameters. However, this type of study may provide an indication of the relationship between parameters based on statistical evaluation. The first approach has the advantages of design and control of the experiments and better measurement uncertainty, however, it is restricted to the experimental fit of data of the particular structure. Neither of these approaches provides a means to explicitly predict the average annual indoor radon concentration from the short term-testing results, but rather each provides indications of the trends of annual indoor radon concentrations when other parameters are considered. An approach that provides an explicit relationship between the short-term testing and the actual annual indoor radon concentration has not been developed.

In this section, an analytical method will be utilized to relate the annual indoor radon concentration to the short-term testing results. A mathematical framework is developed, based on the mass balance equation of radon in the indoor environment which facilitates semi-analytical predictions of annual indoor radon levels. This treatment can serve as a tool to establish the general relationship, based on the actual physical processes of entry and removal, between the time-averaged annual level of indoor radon concentration and the short-term testing of the indoor radon concentration.

APPROACH

The general linear differential equation governing the generation and removal of radon gas in the indoor environment can be developed from the mass balance equation of the indoor radon accumulation rate as,

$$\frac{dC(t)}{dt} = S(r,t) - RE(r,t) \tag{1}$$

where C(t) is the radon activity concentration in a differential volume dV (Bq/m^3), S(r,t) is the radon source term in (Bq/m^3s) which mostly consists of radon entry from the sub-structure area, and RE(r,t) represents the radon activity removal term (Bq/m^3s). The latter term includes radon removal by ventilation and radon removal by radioactive decay. The differential form of this equation utilizing spatial integration over the differential volume dV can be written as (Al-Ahmady 1995),

$$\frac{dC(t)}{dt} + \left(\frac{Q(t)}{V} + \lambda\right) C(t) = \frac{R(t)}{V} \tag{2}$$

where Q(t) is the structure ventilation rate (m^3/s), R(t) is the total radon entry rate (Bq/s), λ is the ^{222}Rn decay constant, and V is the indoor volume (m^3). Utilizing the integrating factor method, the product of the integrating factor and the solution C(t) form an exact differentiation as,

$$\frac{d}{dt}\left[e^{\int\left(\frac{Q(t)}{V} + \lambda\right)dt} C(t)\right] = \frac{R(t)}{V} e^{\int\left(\frac{Q(t)}{V} + \lambda\right)dt} \tag{3}$$

the general solution to Equation 2 is,

$$C(t) = \frac{\int e^{\int\left(\frac{Q(t)}{V} + \lambda\right)dt} \frac{R(t)}{V} dt + C}{e^{\int\left(\frac{Q(t)}{V} + \lambda\right)dt}} \tag{4}$$

This equation represents the expected value of indoor radon concentration (Bq/m^3) at a particular point of time (time stamp) that is measured from a specific time when the solution was performed using a specific initial condition of indoor radon concentration. The time stamp of the initial condition is selected to be t=0 representing the start of the solution application in the indoor environment. For example, if a time period of 1 year (3.1536×10^7 s) is applied from this t=0, the total period of the solution has 3.1536×10^7 time stamps.

The integration terms in Equation 4 need to be evaluated based on indefinite integration and establishing an initial condition which will serve as a reference to the solution in any time stamp point that is located in the range t>0. The constant value C derived by the indefinite integration can be evaluated by using the initial condition. Such evaluation requires solving the integral, and consequently, the relationship between radon entry and ventilation rates with time need to be known. If the latter rates (Bq/s) are constant over the time period of the solution, application of the initial condition, such as $C=C_o$ at t=0 yields,

$$C = -\left[\int e^{\int\left(\frac{Q(t)}{V}+\lambda\right)dt} \frac{R(t)}{V}\, dt\right]_0 + C_0 \left[e^{\int\left(\frac{Q(t)}{V}+\lambda\right)dt}\right]_0 \tag{5}$$

where the straight brackets indicate that integral expression between brackets is to be evaluated indefinitely first, then the value of t is set equal to 0. As seen from Equation 5, the term C in Equation 4 may have an expression as opposed to a number, however, it is constant over the time period of the solution. The evaluated constant term C in Equation 4 is then redefined as C_e indicating that the general solution in Equation 4 contains a known constant that has been evaluated at t=0. The general solution to Equation 2 including the utilization of initial conditions can then be written as,

$$C(t) = \frac{\int e^{\int\left(\frac{Q(t)}{V}+\lambda\right)dt} \frac{R(t)}{V}\, dt + C_e}{e^{\int\left(\frac{Q(t)}{V}+\lambda\right)dt}} \tag{6}$$

Equation 6 represents the time dependent indoor radon concentration that can be evaluated at any point in the time spectrum $t \geq 0$. For each time stamp there is a corresponding value for C(t). For example, the indoor radon concentrations in the structure, where C_e is evaluated at t=0, at 6 months, 9 months, and one year later are then equal to, C(t=1.5552×10^7 s) , C(t=2.3328×10^7 s), and C(t=3.1536×10^7 s), respectively. Indoor radon measurements using any system are based on counting a parameter corresponding to the alpha particles that is converted into a concentration of Bq/m^3. The minimum collection time is 1 second. If the system is designed to provide time dependent measurements, then outputs can be reported per time interval that may vary in length or be selected by the user. If the system provides time-integrated measurements, then one value representing the average indoor radon concentration can be extracted at the conclusion of the testing. In both cases, the system accumulates values of the radon concentration in the minimum period of time based on the design.

For the time dependent systems, the reported indoor radon concentration values collected in time periods can be used to calculate the average indoor radon concentration using a simple arithmetic average or a weighing method such as the reciprocal of the measurement variance. In all cases, the reported average indoor radon concentration is the weighing of the accumulated indoor radon measurements recorded in all the time stamps which occurred during the testing. Such a value, in fact, is the average of the integral of C(t) over the testing period,

$$C_{avg} = \langle C(t) \rangle = \frac{\int_0^t C(t)\, D(t)\, dt}{\int_0^t D(t)\, dt} \tag{7}$$

where D(t) is the time distribution of indoor radon concentration over the period of t=0 to t=t. For testing purposes, D(t) is the arithmetic average of readings or the weighted average of readings by the reciprocal of the variance. If the first case is selected, Equation 7 then becomes,

$$C_{avg} = \frac{1}{T} \int_0^t C(t)\ D(t)\ dt \tag{8}$$

where T=t-0, and equal to the total number of time stamps in the testing period. For example, if the testing system is set to calculate indoor radon concentration based on 1 second measurement intervals, and the test was performed from noon the first day to noon the third day of the seven month after t=0, then the average indoor radon concentration reported by the measurement is,

$$C_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} C(t)\ dt$$
$$= \frac{1}{1.5768x10^7 - 1.55952x10^7} \int_{1.55952x10^7}^{1.5768x10^7} C(t)\ dt \tag{9}$$

where there is $1.728x10^5$-time stamps of 1 second during this 48-hour testing period.

PROCEDURE DEVELOPMENT

A measurement time interval may be reported for a period greater than one second by using a simple conversion. If the measurement is reported in a one hour period, then the previous example will contain 48 readings and the averaging will be performed on the 48-hour testing period. However, conversion will need to be made during evaluation of the integral in order to maintain the correct units since the components of C(t) are the radon entry and removal rates which are reported in Bq/s.

It is important to note that the following treatment provides means not only to estimate the averaged indoor radon concentration over the testing period but also to relate the latter to the particular time of the year. To facilitate this capability, the ratio of the indoor radon concentration reported by a testing period, $(C_{avg})_{ST}$, performed during a particular time in the year (T_x, T_y) to the average indoor radon concentration over the whole year, $(C_{avg})_{LT}$, can be estimated as,

$$\frac{(C_{avg})_{ST}}{(C_{avg})_{LT}} = \frac{\frac{1}{T_y - T_x} \int_{T_x}^{T_y} C(t)\ dt}{\frac{1}{T} \int_0^T C(t)\ dt} \tag{10}$$
$$T > T_y > T_x$$

where T is the long-term length of time (one year for annual estimation), T_y is the time stamp representing the end of the short-term testing period, and T_x is the time stamp representing the beginning of the short-term testing period.

Equation 10 establishes the relationship between the average indoor radon concentration measured over a short-term testing period in a random segment during the year and the time-averaged annual indoor radon concentration. It should be noted that Equation 10 establishes the described relationship independently from how long and when the short-term testing occurs during the year.

If the indoor radon concentration over the year is constant, then Equation 10 will return a value of 1 despite how long and when the short-term testing is performed. For complete evaluation of the equation, the time distribution of indoor radon concentration, particularly the entry and removal rates, needs to be known. If those expressions are incorporated into Equation 10 then,

$$\frac{(C_{avg})_{ST}}{(C_{avg})_{LT}} = \frac{\frac{1}{T_y - T_x}\int_{T_x}^{T_y}\left[\frac{\int e^{\int(\frac{Q(t)}{V}+\lambda)\,dt}\frac{R(t)}{V}\,dt + C_e}{e^{\int(\frac{Q(t)}{V}+\lambda)\,dt}}\right]_{T_x}^{T_y} dt}{\frac{1}{T}\int_0^T\left[\frac{\int e^{\int(\frac{Q(t)}{V}+\lambda)\,dt}\frac{R(t)}{V}\,dt + C_e}{e^{\int(\frac{Q(t)}{V}+\lambda)\,dt}}\right]_0^T dt} \quad (11)$$

The general approach to applying the relationship above between the short-term testing result and annual indoor radon concentration is to estimate the right hand side of Equation 11 using the previously developed expressions for radon removal and entry rates. After the right hand side of Equation 11 is evaluated, the long-term annual indoor radon concentration can be calculated from,

$$(C_{avg})_{LT} = \frac{(C_{avg})_{ST}}{RHS} \quad (12)$$

where RHS is the numerical value of the right hand side of Equation 11.

The best results of estimating this ratio are obtained by performing the integration when the time distribution functions of R(t) and Q(t) are available. The latter case corresponds to a time interval that asymptotically approaches zero. Actual measurement must have a time interval greater than zero and the minimum known interval is 1 second. Therefore, estimations of the integral of Equation 11 can be accurately performed using the actual value of the variable (R and Q) at each of the time steps. The time step is therefore the actual time interval used during the measurement. Smaller time intervals will produce better results.

The discrete nature of the variables in the above mathematical framework allow an estimation of the integral of indoor radon concentration as,

$$\int_{T_x}^{T_y} C(t)\ dt \equiv \sum_{i=T_x}^{T_y} C_i \quad (13)$$

where i is an incremental index representing the time stamp where a measurement value is reported. Incorporating the integral equivalent in Equation 13 into Equation 10 yields,

$$\frac{(C_{avg})_{ST}}{(C_{avg})_{LT}} = \frac{\frac{T}{T_y - T_x}\left[\sum_{i=T_x}^{T_y} C_i\right]}{\left[\sum_{i=0}^{T} C_i\right]} \quad (14)$$

The time term of the above equation is known. Components which drive the indoor radon concentration need to be known (measured) during the long-term period to be estimated in a time interval equal to the time interval of the short-term measurement performed during a particular time of the year in that year.

Indoor radon concentration (C_i) expressions needed for the previous equation have been developed for different types of structures utilizing mechanistic and empirical models either as a whole or from integrated components. Some of the components may be restricted to the particular structure especially where elements characterizing that structure are considered. For the purpose of applying Equation 14, component models for the indoor radon concentration at the University of Florida Radon Research House (UFRRH) will be utilized. The general steady state indoor radon concentration can be represented by,

$$C = \frac{R}{Q + \lambda\ V} \tag{15}$$

where R is the total radon entry rate (Bq/s), Q is the structure ventilation rate (m^3/s), V is the structure indoor volume (m^3), and λ is the Rn-222 radioactive decay constant (1/s). The total radon entry rate can be calculated from convective and diffusion entry from the sub-structure area, R_{conv} & R_{diff} respectively, and convective entry from the ambient R_{amb} (Hintenlang and Al-Ahmady 1994). Convective entries in R are derived by pressure differentials developed by changes in barometric pressure, temperature differences, wind blowing on the structure, and household appliances such as the heating, ventilating, and air-conditioning (HVAC) systems (Al-Ahmady 1995). The structure ventilation rate is also derived by the pressure differential generating mechanisms. For this application indoor radon driving force models of temperature differences (Al-Ahmady and Hintenlang 1994 a&b), barometric pressure variations (Al-Ahmady 1992, Hintenlang and Al-Ahmady 1992), wind blowing on the structure (Al-Ahmady 1995), and HVAC system (Hintenlang and Al-Ahmady 1994) will be utilized. These models, consequently, are

$$\Delta P = P_o\ D\ [(T_1 - T_2)/T_1 T_2] \tag{16-a}$$

$$\Delta P_{slab} = (P_b(t + \Delta t) - P_b(t))\ \exp(-\Delta t/T_R) \tag{16-b}$$

$$\Delta P = C_w\ (P_{in} - P_b - 0.5\ \rho\ v^2) \tag{16-c}$$

$$\Delta P = \exp[(1/n)\ \ln(Q/K)] \tag{16-d}$$

where P_o is the barometric pressure at a common reference level in the structure (Pa), D is a constant equal to 0.0477, T is the absolute temperature (K), Pb is the barometric pressure (Pa), T_R is the characteristic equilibrium time (s), the time required for the spatial sub-slab soil location to equalize with the change in barometric pressure, C_w is the drag coefficient constant (0-1), P_{in} is the indoor pressure (Pa), v is the directional velocity of the wind stream around the structure (m/s), Q is the volumetric flow rate into or out of the structure (m^3/s), n is the flow exponent and has a value between 0.5 to 1, and K is the flow coefficient for the structure. K and n need to be empirically determined using a best linear fit for the testing result per the particular structure. For the UFRRH, these values are 0.0566 and 0.69 for the flow coefficient and flow exponent, respectively (Al-Ahmady 1995).

The total radon entry rate for the UFRRH (Hintenlang and Al-Ahmady 1994) can be calculated from

$$R = 13.23\ \Delta P_{slab} + 1.075\ (\Delta P_{in/out})^{0.69} + R_{diff} \tag{17}$$

where $\Delta P_{in/out}$ is the pressure differentials between the indoors and outdoors. The total radon entry during the short-term testing can be evaluated as,

$$R = (\frac{1}{n\delta t}\ [13.23 \sum_i^n \Delta P_{slab} + 1.075\ (\sum_i^n \Delta P_{in/out})^{0.69}) + R_{diff} \tag{18}$$

where δt is the measurement time interval, and n is the number of measurements during the testing period. The same treatment is utilized for the long-term period (1 year) but n is replaced by m which represents the number of measurements during the year. Using the expressions of the pressure differentials generating mechanisms of Equations 16-a to 16-d, radon convective entry from the sub-slab area is then,

$$R_{conv}^{ss} = 13.23\sum_i [\ ([P_b(i+\Delta i)-P_b(i)]\,e^{(-\frac{\Delta i}{T_R})} + C_w(P_{in}(i)-P_b(i) + 0.5\rho v(i)^2) + 0.0477P_0(\frac{T_{ss}(i)-T_{in}(i)}{T_{ss}(i)\,T_{in}(i)})\,] \quad (19)$$

Radon convective entry from the ambient may be driven by the temperature difference between the outdoor and the indoor and the wind blowing that causes interior depressurization. Therefore this entry is calculated by,

$$R_{conv}^{amb} = 1.075\sum_i [\ C_w(P_{in}(i)-P_b(i)+0.5\rho v(i)^2) + 0.0477P_0(\frac{T_{out}(i)-T_{in}(i)}{T_{out}(i)\,T_{in}(i)})\,]^{0.69} \quad (20)$$

The radon removal rate contributes to the reduction of indoor radon concentrations and is derived from the temperature difference between the indoors and the outdoors and wind blowing on the structure which pressurizes the interior. The radon removal rate for this application can be calculated by the same manner as the radon entry rate as,

$$Q = 0.0566\sum_i [\ C_w(P_{in}(i)-P_b(i)+0.5\rho v(i)^2) + 0.0477P_0(\frac{T_{in}(i)-T_{out}(i)}{T_{in}(i)\,T_{out}(i)})\,]^{0.69} \quad (21)$$

Radon diffusive entry is independent from the above driving forces needs to be estimated for this application. If Equations 19, 20, and 21 are integrated into Equation 14, the fundamental expression relating the short-term time-integrated measurement of indoor radon concentration and the long-term annual time-averaged indoor radon concentration is produced. A year's worth of temperature, wind speed, and barometric pressure measurements are needed for this application. Application of the above mathematical framework was performed to estimate the annual indoor radon level during one year period May 1991 to May 1992 at the UFRRH and compare it with the short-term testing performed under neutral pressure conditions. The predicted time-averaged indoor radon concentration is 1539.2 Bq/m^3 (41.6 pCi/l), while six short-term tests produced results that differ from the estimated annual value by a maximum factor of approximately 1.6.

Data required to perform estimation of the time-averaged annual indoor radon concentration are available from the National Weather Information Center operated by the Federal Aviation Administration. All the above data are collected on an hourly basis in most of the cities at the airport locations. The diffusive component of radon from the sub-structure area into the indoors can be estimated or ignored if the convective flow is dominant, depending on the soil properties and structural features. Indoor and sub-slab temperatures can also be estimated if they are not available, depending on the general weather pattern in the area.

CONCLUSIONS

The mass balance approach can be used to establish the relationship between the short-term indoor radon concentration measurement and the time-averaged annual indoor radon concentration measurement. This approach developed the first mathematical framework needed for such an application and needed to support an explicit semi-analytical treatment that accounts for all radon entry and removal driving forces in the indoor environment. A semi-analytical framework can be developed by integrating the semi-diurnal barometric pressure variation driven pressure differences across the slab, temperature-induced pressure differences across the structure slab and shell, and wind-induced pressure differences across the slab and the structure walls into the differential pressure formulation of the total radon convective entry from the sub-structure area into the building interior. Radon removal driving forces consisting of the models of structure ventilation and accounted for in the decay of radon in the indoor environment are also integrated into the time-dependent indoor radon concentration derived from the mass balance principle. The latter mathematical structure can then be applied to the short- and long-term averaged indoor radon concentration estimation in two different time periods when a common reference time is established. The ratio of the short-term indoor radon concentration to the long-term time averaged indoor radon concentration can be evaluated for all configurations of testing periods and the relative occurrence of the short-term test with respect to the long-term period.

Utilization of such applications to predict the annual averaged indoor radon concentration requires the possession of a year's (or more) worth of environmental data such as temperature, barometric pressure, and wind speed. These data are integrated into the calculation process using the concept of time indexing to facilitate the solution. Such data are available from the National Climate Data Center which report the Federal Aviation Administration weather measurements. Most of the above data are collected on an hourly basis in most large cities at airport locations. The diffusive component of radon from the sub-structure area into the indoor can be estimated or ignored if the convective flow is dominant, depending on the soil properties and structural features. Indoor and sub-slab temperatures can also be estimated if not available, depending on the general weather pattern in the area. This application provides a significant tool to predict the time-averaged annual indoor radon concentration used for health hazard estimations and legal considerations from the wide spread practice of short-term (48-hour) testing of indoor radon concentration.

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