

## VENTILATION LEVEL FROM TIME PROFILE OF RADON

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### INTRODUCTION

In this paper I will explain how the data from a continuous radon monitor can be used to estimate the minimum level of ventilation for a home. It is well known that radon levels in a home follow a sinusoidal pattern. This is simply explained by noting that the strength of the radon source is proportional to the indoor-outdoor pressure difference which itself follows a sinusoidal pattern due to solar diurnal heating. The magnitude of the radon concentration is inversely related to the ventilation level, and the peak of concentration will lag (follow behind) in time the peak amplitude of the source according to the ventilation rate in the home.

Conditions for a screening test are designed to reveal the reasonable worst case level for a home. The important factor in this is to keep the house in minimum ventilation or "closed-house" condition. The doors and windows are closed except for normal entrance and exit and the occupants are requested to close the doors immediately when leaving or returning. These are not extreme restrictions because they naturally occur in homes during cold or otherwise inclement conditions. The ASHRAE Standard 62-1989 specifies that residential buildings should have a minimum of 15 cfm per person or an air-change rate of 0.35 ACH (air changes per hour). The latter level is usually the appropriate standard, except for large families in small homes. For example, my home has a volume of approximately 20,000 cubic feet (three bedroom, ranch style home). A ventilation rate of 0.35 ACH corresponds to about 120 cfm which is about 30 cfm for each of the four residents. Using dry-ice to load the empty house with CO<sub>2</sub>, we have found that our home has a minimum, closed-house, exchange rate of about 0.6 ACH or 200 cfm. Clearly, this exceeds the minimum ASHRAE requirement. Note my location in Santa Barbara has a mild climate, rarely going below 50°F or above 90°F. I believe, however, that most homes in US do satisfy the ASHRAE requirement; only the most stuffy homes are suspect.

In this paper I focus on residential buildings exclusively. This is done because most residential buildings do not have active ventilation systems to bring in outside air. The heating or cooling systems operate strictly on recirculation. The minimum fresh-air ventilation results from extraneous infiltration/exfiltration from the numerous unavoidable cracks in the building envelope. Important reasons to consider homes is that people spend most of their time there and it is also where most radon testing occurs.

Sufficient ventilation in a home is important for health reasons. There are numerous sources of air contaminants in a home, for example, cleaning fluids, pets, synthetic materials, cooking odors, and bathroom smells. The introduction of fresh air is needed to dilute these irritants to acceptable levels. There are various chronic ailments associated with poor ventilation such as headaches, eye irritation and feelings of malaise.

In the following section the mathematical model for ventilation is summarized and discussed. Some more details are included in an appendix. The succeeding section provides some actual data taken from homes. Lastly, the conclusions are summarized.

## MATHEMATICAL MODEL

The time rate of change of the radon concentration  $C(t)$  in a house is approximately given by an ordinary first-order differential equation:

$$\frac{dC(t)}{dt} = S_V(t) - A(t)C(t) \quad (1)$$

where  $S_V$  is the intensive source for radon radioactivity per unit time per unit volume [ $S_V = S$  (pCi/hour into house)/ $V$ -house(liters)] and  $A$  is the air-exchange rate for the house (per hour, the units are designated ACH or simply  $h^{-1}$ ). All of the variables are time dependent and positive definite. The primary assumption is that the radon gas mixes instantly throughout the air volume of the house. The time scale for the gas equilibration within a closed house is only a few minute so this is justified. Also, recall that we are applying the analysis to a situation of minimum air exchange so there will not be significant airflows to entrain the radon and thereby maintain inhomogeneous distributions. The outside air is assumed to be radon free. Recirculation flow within the house will enhance the mixing rate. Other model details are discussed in the appendix.

The solution for the concentration is written:

$$C(t) \exp(\int A dt) = \int [S_V \exp(\int A dt)] dt + K \quad (2)$$

where  $K$  is the constant of integration, e.g. the radon concentration at time  $t=0$ . As will be shown, the most appropriate approximation for this equation is when the ventilation rate is constant  $A(t)=A_0$  and the source has a simple sinusoidal variation  $S_V(t)=S_0+S_1 \cos(\omega t)$  where  $\omega=2\pi/24 h^{-1}$  the angular velocity of a day. Substituting into Eqn. (2) and evaluating

$$C(t) = \frac{S_0}{A_0} + \frac{S_1}{\sqrt{A_0^2 + \omega^2}} \cos(\omega t + \phi) + K e^{-A_0 t} \quad (3)$$

where phase  $\phi$  is defined by  $\tan(\phi) = \omega/A_0$ . For the assumed sinusoidal variation the integration constant  $K$  reflects any initial solution mismatch which will exponentially decay with an e-folding time of  $1/A_0$  hours..

The first term is the equilibrium radon concentration  $S_0/A_0$  that would result for a constant source in a house with constant source and ventilation. This can be seen by setting the

time variation to zero in Eqn. (1). The second term has the same form as the source [e.g., sinusoidal] except that it is shifted to the right by phase  $\phi$ . The following table shows this phase as a function of the air-change rate and the resulting delay in hours.

$A(\text{h}^{-1})$	$\omega/A$	$\phi$ (radians)	$\Delta t$ (hr)
0.35	0.748	0.642	2.45
0.5	0.524	0.482	1.84
1.0	0.262	0.256	0.978
2.0	0.131	0.130	0.497

Radon has a decay constant of  $\lambda=0.00753 \text{ h}^{-1}$ , which is irrelevant because the time scale for the air-exchange is so much larger. The constant  $A_0$  may be thought of as including the radioactive decay;  $\lambda$  is the lower bound for  $A_0$ . For very small  $A_0$ ,  $\phi \rightarrow \pi/2$  and  $\Delta t \rightarrow 6 \text{ hr}$ ; the radon concentration is then approximated by  $\sin(\omega x)$  time variation.

For a drafty houses (very large  $A_0$ ), the radon levels are small. For large air exchange the concentration time delay is approximately  $1/A_0$ . The phase angle  $\phi$  is small and the radon concentration follows the source time pattern, approximately as  $\cos(\omega x)$ .

As discussed in the appendix, the uncertainty in the amplitude and phase of the radon concentration can be approximated using statistical methods.

## RADON DATA

Figure 1 shows the radon counts for a continuous monitor each half hour beginning at noon on December 16, 1993, and continuing for about 18.5 days. The detector background of about 5 counts has not been subtracted from these measurements. The monitor was placed in the middle of the dining room table. For two weeks beginning the evening of Dec. 17, I was on vacation with the house closed, but even after I returned the measurements continue with the same pattern. There was no weather of note during the measurement period. (There had been a little rain before the beginning, and I have since learned that drying soil will tend to hold soil gases resulting in these lower measurements near the beginning.) The calibration for the monitor is such that there will be about 10 counts for each pCi/L of radon gas. The average radon concentration during this period was 1.9 pCi/L with a minimum of 0.0 pCi/L and a maximum of 4.4 pCi/L.

Figure 2 shows the Fourier transform of the data in Figure 1 with frequency of inverse days from  $0.1 \text{ day}^{-1}$  to  $10 \text{ day}^{-1}$ . As you can see, there is a pronounced resonance for the period of one day. Other than the average value there is only one statistically non-zero Fourier component. There is a considerable amount of noise in these measurements. For these low concentration measurements, the uncertainty is about 5 counts per half hour

(0.5 pCi/l). After 18 days, the uncertainty for a given half-hour average is reduced to 1.2 counts. (There are a total of about 400 counts in each half hour of the day, with a standard deviation of 20 counts; e.g., the period from 10-10:30 AM was measured each day for 19 consecutive days with 424 counts, mean=22.3, standard error of mean=1.3)

The relatively long measurement period was necessitated by the need to achieve good statistics for each of the half-hour periods during the day. Radioactive decays follow a Poisson distribution with the mean equal to the variance. If you want a measurement average that is within about 5% of the true average, you need to accumulate on the order of 400 counts. Therefore, if the source has sufficient magnitude, a long measurement would not be needed.

In Figures 3 two similar radon profiles are shown. The first is for my dining room as discussed above. The scattered data points show the average counts in each of the half hour periods. The second is a measurement of a empty house located on a radioactive geologic formation for two days. The average counts per half hour are significantly larger, but the total counts, and therefore the statistics, are about the same. The dining room data begins at 12:15 PM while the radioactive formation data started at 10 AM. The peak of both these curves occurs at about 6:30 AM with an uncertainty in phase of about 20 min. Both measurements are well characterized by a constant and a single Fourier component (sine wave) with a period equal to one day.

If we assume that my measurement of the air change rate of  $A_0=0.6$  ACH is valid, then these radon concentration curves will lag the source curve by 2.5 hours. The source would achieve its maximum level about 4 AM which is not unreasonable. Figure 4 shows the inferred curves for the radon source and concentration. The counts have been converted into the standard units of pCi/L/hr for the source and pCi/L for the concentration. The times have been arranged to show the day period with the time beginning at noon.

## SUMMARY

Usually houses do not have ventilation systems to actively bring in fresh air. But the occupants need a minimum amount for health reasons. A radon test is designed to estimate the reasonable worst case at minimum ventilation for the building. The question arises: can the radon measurements be used to determine the minimum ventilation level? We have shown that the radon concentration in the building will lag behind the source time profile because of the ventilation. The amount of time lag is inversely related to the magnitude of the ventilation. We need to know two things: (1) the time profile of the radon source rate, and (2) the time profile of the indoor radon concentration.

## APPENDIX - Analysis Details

Here we provide some information about the analysis: (1) the basic equation and its assumptions; (2) the Fourier analysis; and (3) the statistics.

Basic response of air in closed house with a pollutant source--Suppose you have a closed house, minimum air exchange rate  $A(t)$ , with a volume  $V$  (L) and a time dependent source of radon gas  $S(t)$ . In air pollution the amount of a foreign gas is given by its mass, but for radon the mass appears to be negligible so the amount is characterized by its radioactivity (pCi) which is proportional to its mass. The source has units of pCi/hr.

Although the house is closed, it is certainly not air tight and there is continual exchange with the outside air given by some volumetric flow  $Q(t) = A(t) \cdot V$ . The units of  $Q$  are volume per unit time (L/hr). (Volume is also represented by cubic feet, 1.00 cubic foot = 28.3 L.) For air exchange of reasonable magnitude,  $A(t) < 2.0$  ACH, the radon radioactivity of the indoor air is approximated by

$$\frac{dR}{dt} = S(t) - A(t)[R(t) - R_{out}].$$

The reason for limiting the air-exchange is to give the radon a chance to uniformly mix in the indoor air volume. Even if the source is in one corner of the house, the resulting gas will quickly mix throughout the house on a timescale of minutes. (A ventilation rate of  $A=2$ , has a time constant of  $1/A = 0.5$  hr = 30 min.) In generally I expect an unlocalized source distributed over the floor area. Since the airflow is separate, this equation seems to imply some uranium (and therefore radium) emanating the radon within the building.

If all the terms in the above equation are divided by the house volume, the equation gives the concentration of the radon

$$\frac{dC}{dt} = S_v(t) - A(t)[C(t) - C_{out}].$$

where  $C_{out}$  is the radon concentration in the ambient (outside) air and  $S_v$  is the indoor volume intensive source strength (units of pCi/L/hr). This means  $R_{out}$  in the above equation is the total radon radioactivity in the outside air in a volume equal to the house. The ambient radon concentration is assumed to be negligible  $C_{out} = 0$ .

Like the source, the air exchange is conceptualized as being dispersed about the envelope of the house. Often houses will have some localized areas of increased ventilation. Examples are doggie doors, chimneys, sky lights, and plumbing vents. Indeed, any of these may dominate the incidental ventilation. By assuming uniform mixing, my math model is more consistent with a general seeping or diffusing of air throughout the building envelop. Houses are typically rather leaky buildings. In fact, since there is usually not an system to bring in fresh air, the occupants depends on the leaks for fresh air. There is always an upward convection of air within a house because of pressure differences. Air is drawn in around the floor by a negative pressure (inside less than outside), is warmed within the building, and expelled around the ceiling by a positive pressure. The resulting flows are definite, but slow. The air in the house maintains an overall equilibrium pressure with the ambient which means the floor inward flow is closely balanced by the ceiling outward flow. Thus, the time functions of the source  $S_v$  and the ventilation  $A$  are about the same since they are both driven by the indoor-outdoor pressure difference.

The reason we are interested in radon is because it is radioactivity and the damage it does to living tissue. The radioactivity is characterized by a decay time or half life which acts as a sink for the radon. Whenever it decays, it vanishes. (In this paper we do not consider the behavior of the radon daughter products.) To account for decay the equation is modified

$$\frac{dC}{dt} = S_v(t) - A(t)C(t) - \lambda C(t)$$

The half life for radon is  $t_{1/2} = 92$  and its decay constant is  $\lambda = \ln(2)/92 = 0.00753 \text{ h}^{-1}$ . Since it is the most massive of the noble gases, radon does have some electronegativity and some chemistry. Although the chemistry of radon has not been studied extensively, it is believed to be very slow. A very stuffy home will have an air change rate of at least  $0.1 \text{ h}^{-1}$  which is large compared to the decay rate or chemistry. These other sinks for radon are so small they are lost in the uncertainty associated with the ventilation. The air-exchange constant is therefore generalized to be a general loss constant for the radon and include the slow radioactive decay and any chemistry. (The losses due to chemistry may be the same magnitude as the nuclear decay.)

A general solution to the remaining equation is written by using an integrating factor. Both sides are multiplied by the exponential of the integral of  $A(t)$ ,

$$\left[ \frac{dC}{dt} + A(t)C(t) \right] \exp\left(\int A dt\right) = S_v(t) \exp\left(\int A dt\right)$$

This could be the integral of  $A$  from time zero to the present. Any initial condition on the ventilation is becomes a non-zero constant multiplying both sides. The left-hand side is now a total derivative,

$$\frac{d}{dt} \left[ C(t) \exp\left(\int A dt\right) \right] = S_v(t) \exp\left(\int A dt\right)$$

with the immediate solution

$$C(t) \exp\left(\int A dt'\right) = \int \left[ S_v(t') \exp\left(\int A dt'\right) \right] dt' + K$$

where  $K$  is a constant of integration, the indoor radon concentration at time zero, and I have added a prime to the inner differential to avoid confusion. The solution for  $C(t)$  is given by

$$C(t) = \exp\left(-\int A dt'\right) \int \left[ S_v(t') \exp\left(\int A dt'\right) \right] dt' + K \exp\left(-\int A dt'\right)$$

Since all the quantities are positive definite, the value of the radon at time zero is an initial condition whose influence will inexorably die away. Consider, for example, the solution for a constant source  $S_0$  with a constant level of ventilation  $A_0$ . It is

$$C(t) = \frac{S_0}{A_0} + \left( C_0 - \frac{S_0}{A_0} \right) e^{-A_0 t}$$

If the initial concentration  $C_0$  is not equal to the equilibrium value  $S_0/A_0$ , then the mismatch concentration will exponentially decay (either rising or falling) to  $S_0/A_0$ . This is characteristic of the time profile that will result for a step function change in the source. For example, when a clothes dryer is activated, it will cause a step function pressure decrease on the floor usually drawing up a constant rate of radon. The radon in the house will increase according to  $[1 - \exp(-A_0 t)]$  and then, when the dryer ends its cycle, the radon will likewise be diluted according to an exponential function with the same time constant.

Changing the level of ventilation in step-function fashion will also result in profile like the above with appropriate values inserted. For example, suppose there is a house at some equilibrium radon concentration  $S_0/A_0$ , then a weather front comes through changing the pressure relations so that there is a new source level  $S_1$  and new air exchange rate  $A_1$ . The solution for this is

$$C(t) = \frac{S_1}{A_1} + \left( \frac{S_0}{A_0} - \frac{S_1}{A_1} \right) e^{-A_1 t}$$

where the zero time is the time of the weather change. I have often observed exponential rises and falls associated with step changes in radon sources or air exchange rates.

My observations are that the time pattern of radon for closed houses generally follows a sinusoidal profile. With this in mind I consider the concentration equation with a sinusoidal source term,  $S_v(t) = S_0 + S_1 \cos(\omega t)$  with  $\omega = 2\pi/24 \text{ h}^{-1}$  with the air exchange rate constant  $A_0$ . (Note:  $S_1 < S_0$ .) Substituting, the solution is

$$C(t) = \exp(-A_0 t) \int [S_0 + S_1 \cos(\omega x)] \exp(A_0 t) dt + K \exp(-A_0 t)$$

and the integral is easy to evaluate

$$C(t) = \frac{S_0}{A_0} + S_1 \left( \frac{A_0 \cos \omega x + \omega \sin \omega x}{A_0^2 + \omega^2} \right) + \left( C_0 - \frac{S_0}{A_0} - \frac{S_1 A_0}{A_0^2 + \omega^2} \right) e^{-A_0 t}$$

The solution can be made a bit more understandable, by writing it as a single sinusoidal function using a phase angle  $\phi = \text{atan}(\omega/A_0)$ .

$$C(t) = \frac{S_0}{A_0} + \frac{S_1}{\sqrt{A_0^2 + \omega^2}} \cos(\omega x + \phi) + \left( C_0 - \frac{S_0}{A_0} - \frac{S_1 \cos \phi}{\sqrt{A_0^2 + \omega^2}} \right) e^{-A_0 t}$$

As you can see, this is a single sinusoidal wave that lags behind the source function with a phase that is inversely related to the magnitude of the air exchange rate. When  $A_0$  is large the phase angle  $\phi$  is small, and vice versa ( $0 < \phi < \pi/2$ ).

A solution can also be obtained for a sinusoidal air-exchange rate  $A(t) = A_0 + A_1 \cos(\omega t)$ . It is a power series in the parameter  $(A_1/A_0)$ ,

$$C(t) = \frac{S_0}{A_0} \left[ 1 - A_1 \left( \frac{A_0 \cos \omega x + \omega \sin \omega x}{A_0^2 + \omega^2} \right) \right] + \left[ C_0 - \frac{S_0}{A_0} \left( 1 - \frac{A_1 A_0}{A_0^2 + \omega^2} \right) \right] e^{-A_0 t}$$

or as above using the phase angle  $\phi = \text{atan}(\omega/A_0)$ ,

$$C(t) = \frac{S_0}{A_0} \left[ 1 - \frac{A_1}{\sqrt{A_0^2 + \omega^2}} \cos(\omega x + \phi) \right] + \left[ C_0 - \frac{S_0}{A_0} \left( 1 - \frac{A_1 \cos \phi}{\sqrt{A_0^2 + \omega^2}} \right) \right] e^{-A_0 t}$$

Notice that as the magnitude of the ventilation increases, the concentration decreases which makes sense. The solution can be written for sinusoidal variation in both source and sink:

$$C(t) = \frac{S_0}{A_0} + \frac{(S_1 - A_1 S_0 / A_0)}{\sqrt{A_0^2 + \omega^2}} \cos(\omega x + \phi) + \left[ C_0 - \frac{S_0}{A_0} - \frac{(S_1 - A_1 S_0 / A_0) \cos \phi}{\sqrt{A_0^2 + \omega^2}} \right] e^{-A_0 t}$$

This solution can also be obtained by linearizing the variables and using Fourier analysis. Each of the variables is assumed to equal some constant plus a sinusoidal function; e.g.,

$$C(t) = C_0 + \int C_\omega e^{i\omega x} d\omega, \quad S(t) = S_0 + \int S_\omega e^{i\omega x} d\omega, \quad \text{and} \quad A(t) = A_0 + \int A_\omega e^{i\omega x} d\omega$$

Substituting in the original first-order differential equation,

$$i \int \omega C_\omega e^{i\omega x} d\omega = S_0 + \int S_\omega e^{i\omega x} d\omega - (A_0 + \int A_\omega e^{i\omega x} d\omega)(C_0 + \int C_\omega e^{i\omega x} d\omega)$$

Multiplying by  $e^{-i\omega x}$  and integrating  $t$  from minus infinity to plus infinity: the constants are  $0 = S_0 - A_0 C_0$  or  $C_0 = S_0 / A_0$ ,

For the time independent part of the solution, suppressing the integration symbols,

$$i\omega C_\omega = S_\omega - (A_\omega C_0 + A_0 C_\omega) - A_{\omega_1} C_{\omega_2} \delta(\omega_1 + \omega_2 - \omega)$$

The final convolution term of the this part is believed to be small, so

$$(A_0 + i\omega)C_\omega = S_\omega - C_0 A_\omega = S_\omega - S_0 A_\omega / A_0, \text{ so } C_\omega = \frac{S_\omega - S_0 A_\omega / A_0}{A_0 + i\omega}$$

where the constant solution for  $C_0$  has been substituted. This is the same solution as was found using direct integration (There is no initial condition in Fourier analysis.)

$$C(t) = \frac{S_0}{A_0} + \int \left( \frac{S_\omega - S_0 A_\omega / A_0}{A_0 + i\omega} \right) e^{i\omega t} d\omega = \frac{S_0}{A_0} + \frac{S_\omega - S_0 A_\omega / A_0}{A_0 + i\omega} e^{i\omega t}$$

given that both the source and sink are delta functions in Fourier space  $\omega$ .

A physical analogy for the situation is a bucket with a hole being filled by a spigot. The flow rate out through the hole is proportional to the level of water in the bucket. If the flow of water into the bucket and the hole are constant, then the water will rise in the bucket until the pressure (concentration) rises enough so that the outward flow through the hole equals the inward flow:  $C_0 = S_0/A_0$ .

If the inward flow has a sinusoidal variation, then the water level of the bucket will also go up and down with the same period as the spigot but there will be a time lag. The time lag will depend on how quickly the outflow responds to the changing inflow. The peak level in the bucket will follow the peak of the spigot rate. When the spigot rate reaches its maximum, the level in the bucket will still be rising to continue increasing the outflow. When the level rises to the level where the outflow equals the inflow, the level has achieved its maximum and the level will begin to decrease.

Suppose the inward flow the spigot is constant, but that the size of the hole was varying with a sinusoidal time function (a little strange). For this situation, the level in the bucket will be drawn down with the increased hole size and vice versa. Thus, the variation in the magnitude of the water level will go in the opposite direction to the variation in the magnitude of the hole. As the hole increases through the peak in its magnitude, the level of the water in the bucket continues to go down until, again, it reaches the point where the inflow and outflow are equal. The phase relation between the two variations is exactly the same as would be from the spigot flow provided they both have the frequency. They are both given by the e-folding time of the average level of water in the bucket.

I have made several measurements of CO<sub>2</sub> in buildings to estimate their minimum ventilation level. I have never observed a variation which appeared to be related to the time of day. My measurements have included a couple of instances where I have used a block of dry ice and allowed it to sublime. In an empty building this will produce a rather long period of high CO<sub>2</sub> levels. The primary methodology for this measurement is to observe the exponential decay of the CO<sub>2</sub> after the block has disappeared so I was not looking for time-of-day effects. Nevertheless, it is my belief that the variation in air-exchange due to solar heating is small but that the effect on the radon source is relatively large. The following are speculation based on my observations:

$$20\% < \frac{S_\omega}{S_0} < 40\% \quad \text{but} \quad \frac{A_\omega}{A_0} < 5\%$$

Recall that these effects work against one another. I see definite daily sinusoidal variation in the radon profiles and believe they are related to the effects of solar heating on the source.

Fourier analysis of radon magnitude--My continuous monitor counts electronic pulses initiated by radon decays and records them each half hour. The instrument is calibrated so that the background level is approximately 5 counts per half hour and 10 counts per half hour corresponds approximately to 1.0 pCi/L. (These are rough numbers; each year my

device is calibrated in a chamber so that the background and response coefficients are estimated to two significant digits.) To satisfy the measurement protocols, the minimum period for a measurement is 48 hours (2 days) which is 96 radon data points  $C_n$ . As discussed in the body of the paper, the only statistically non-zero Fourier component is the one associated with a period of one day. Its coefficients are found using the formula:

$$C_\omega = \frac{1}{48} \sum_{n=1}^{96} C_n [\cos(2\pi n/48) + i \sin(2\pi n/48)]$$

The Fourier coefficients for any period may be found by putting the period of interest as the number of half-hours in the denominator of the argument of the trigonometric functions. The coefficients are equal to half the average of the products  $C_n e^{i\omega t}$  except for the zero frequency (the overall average) and the highest frequency (period = 0.5 hour). Neither of these components has an imaginary part (i.e., they are pure real). There are 96 independent Fourier components associated with the 96 independent pieces of data.

The individual data for each half hour period are dominated by statistical noise. Thus, it is often difficult to visually determine the sinusoidal pattern with much confidence. At a minimum, however, you should be able to see that the magnitude of the measurement in the early morning hours is somewhat higher than those of the early evening hours.

The Fourier mode associated with the 1-day period usually has the largest magnitude. Because of this, I refer to this mode as the fundamental. The fundamental and its harmonics (multiples of the basic frequency) will converge to the average of the signals for each half hour. The average and the fundamental will be the best approximation to the data in the least squares sense. In other words, if a residual is defined as the difference between each of the data points and the analytic value of the function approximation, the sum of these residuals is equal to zero and the sum of the residuals squared is a minimum. Any other value for the coefficients would yield a larger value for the sum of the residuals squared.

Uncertainty in the amplitude and phase of the Fourier fundamental--Because of the limited detector volume, there is a considerable amount of statistical noise in the individual measurements. The signal to noise ratio can be estimated from first principles. It is equal to the square root of the true number of counts. We only have the measured data, but this can give a good approximation for the uncertainty. Consider a detector volume of gas with exactly 4000 atoms of radon within it. They each have a half-life of 92 hours and so they have a decay constant of about  $\lambda = 0.0075 \text{ h}^{-1}$ . In the next half hour an average of  $\lambda n(\Delta t) = 0.0075(4000)(0.5) = 15$  atoms will decay. The number that actually decay will be given by a Poisson distribution with the mean equal to the variance. Thus, the standard deviation of number that decay will be the square root of 15 or approximately 4. This distribution is accurately approximated by a normal distribution so about 68% of the time for this situation, we will measure between 11 and 19 counts. (Note: we could also find the same distribution by noting that the probability of decay in the next half hour for each of the 4000 radon atoms is  $\lambda(\Delta t) = 0.00375$  and using this as a proportion in a binomial distribution.)

For a radioactivity problem, the uncertainty in the number of counts measured is well approximated by the square root of the number of counts measured. The larger the number of counts, the better this approximation (variation on the law of large numbers). If we look at the distribution of residuals, they should not only appear to have a normal distribution, but there should be a clear pattern, else that would mean some other model would provide a better approximation. This is indeed what we observe. When the residuals are plotted, they appear to be random selections from the proper normal distribution. It is clear that the best model for the radon concentration is an average value with a one-day sinusoidal variation.

Statistics of the Fourier fundamental--The uncertainty in the amplitude of the fundamental component may be found by analyzing the count data as above. This means that the times in the morning when the counts are more intense are more accurate. That's right. We can also look at the uncertainty of the amplitude by looking at the square root of average value of the residuals. (To be consistent with standard deviation, divide the sum of the squares of the residuals by n-1 and not n.) We are effectively working with a coefficient of correlation for a sinusoidal approximation to the data. The  $\Delta C$  as given above is the best measure of the error in the magnitude of the amplitude.

We evaluate the phase of the fundamental which is used to represent the number of counts. We can use this to determine our best estimate of the maximum radon counts. To find the uncertainty in this phase we return to its definition:  $\theta = \text{atan}[Im(C)/Re(C)]$  in terms of the coefficients of the sine and cosine of the fundamental. Taking the differential,

$$\Delta\theta = \frac{(\Delta C_R)C_I + (\Delta C_I)C_R}{C_R^2 + C_I^2} \approx \frac{\Delta|C|\sqrt{|C|}}{C_R^2 + C_I^2} = \frac{\Delta|C|}{|C|}$$

which is just the logarithmic differential for the amplitude. This usually indicates that the phase for the response is known to within 30 minutes of the true value.

### Raw Data from Hobbs Dining Room

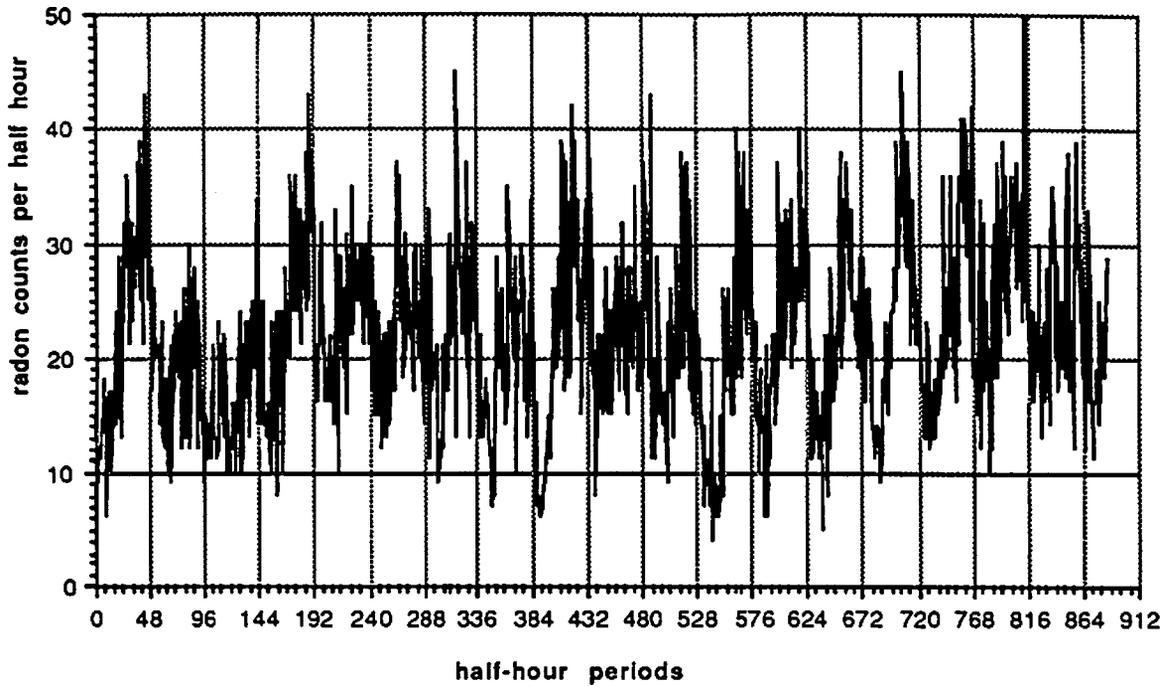


Figure 1. Total radon counts recorded each half hour in Hobbs dining room beginning December 16, 1993. The average measurement corresponds to a radon level of 1.9 pCi/L. The uncertainty for each measurement is  $\pm 0.5$  pCi/L or  $\pm 5$  counts.

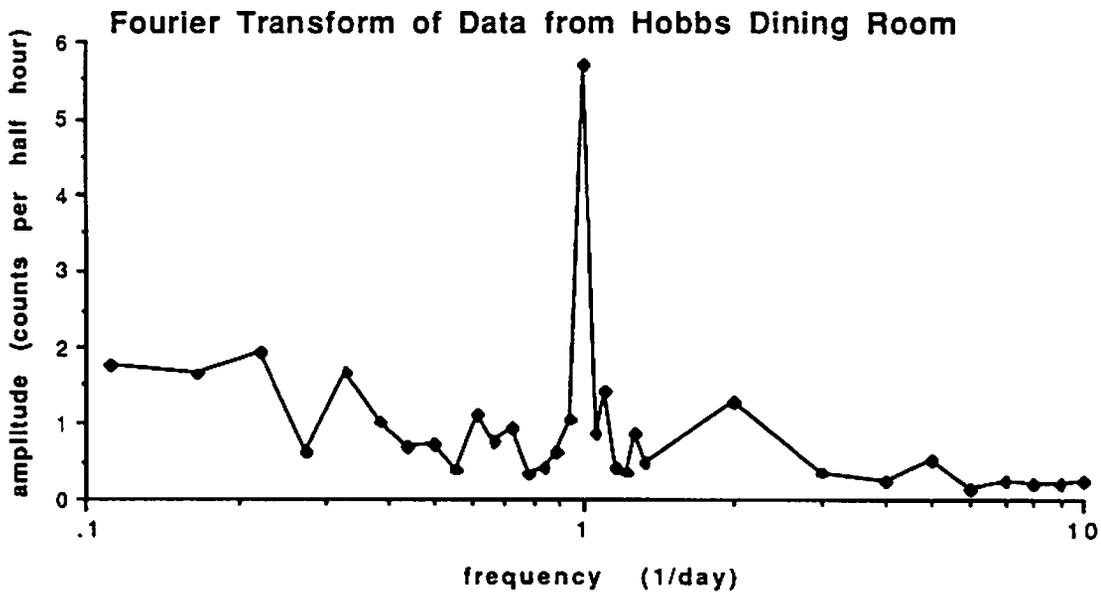


Figure 2. Fourier transform of radon data from Hobbs dining room. There is a strong resonance which occurs for a period of one day. The other modes are consistent with statistical noise.

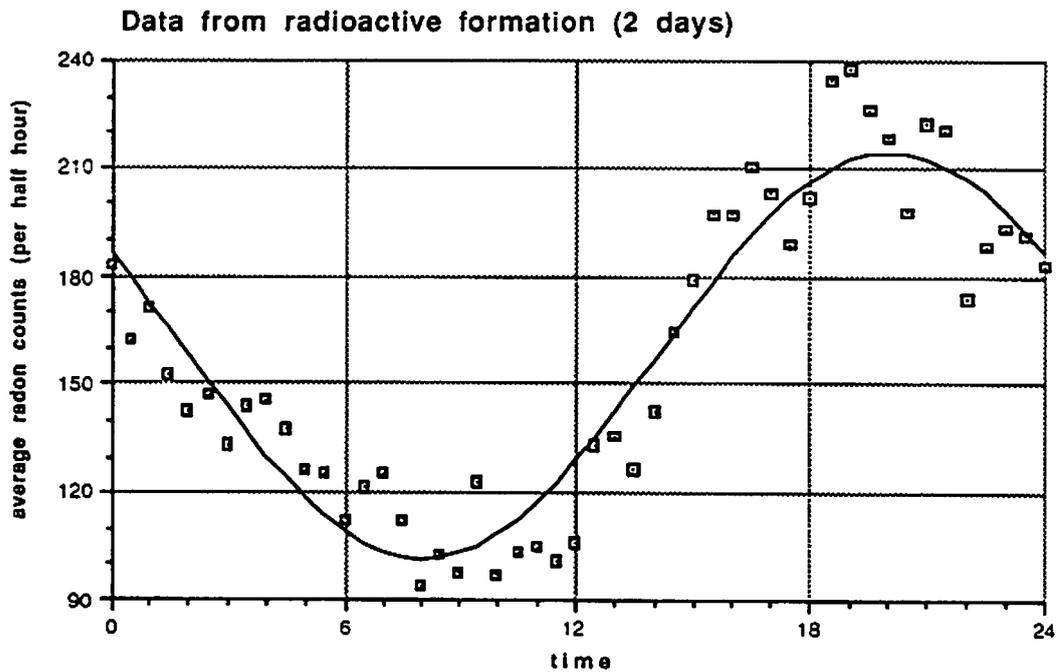
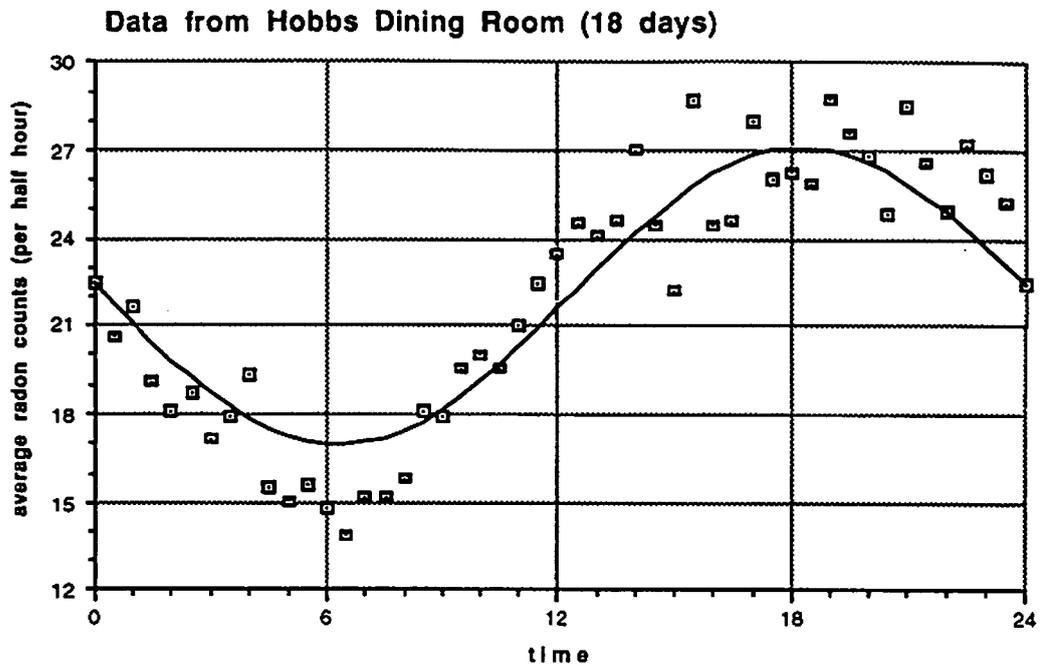


Figure 3. Data and least-squares sinusoidal best fit for two closed houses. A longer period of measurement is required to resolve the variation for a lower magnitude source.

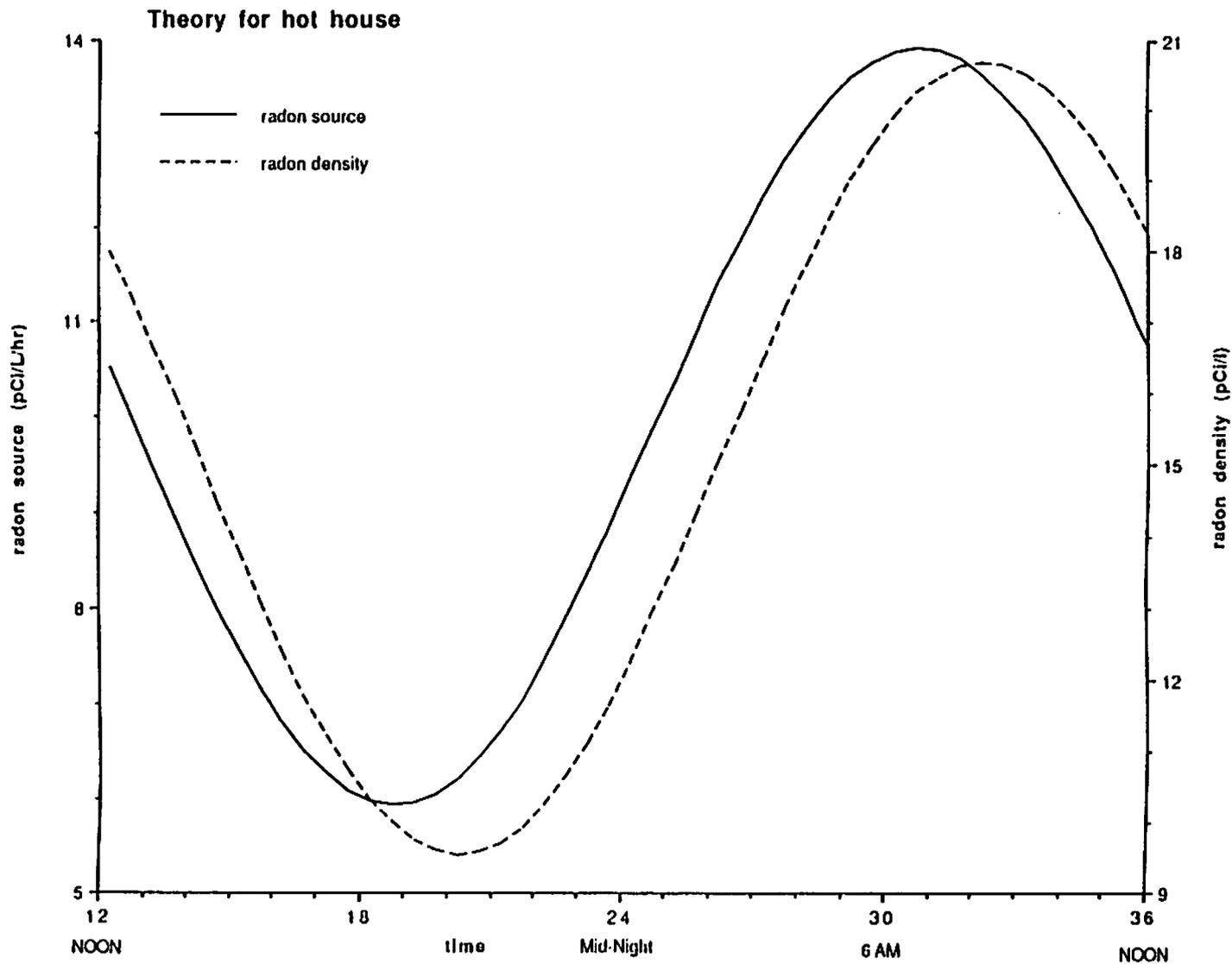


Figure 4. The least-squares best-fit sinusoidal radon density time pattern in radiation density units along with the radon infiltration rate which would cause this pattern. A constant air-exchange rate of  $A = 0.6$  air changes per hour is assumed. This results in the observed approximate 2.5 hour lag of the radon density.